**MATHEMATICS SPECIALIST**

**MAWA Year 12 Examination 2019**

**Calculator-free**

# Marking Key

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The release date for this exam and marking scheme is 14th June.

**Question 1(a) (3 marks)**

|  |
| --- |
| Solution |
| If then Alternatively we note that with and . Then  as before |
| Mathematical behaviours | Marks |
| * calculates  correctly
* calculates correctly

(1 mark for showing real part zero and 1 mark for correct value of imaginary part) | 11+1 |

**Question 1(b) (1 mark)**

|  |
| --- |
| Solution |
| Since then  which is real and negative. Hence |
| Mathematical behaviours | Marks |
| * calculates  correctly
 | 1 |

**Question 2 (a) (1 mark)**

|  |
| --- |
| Solution |
| Augmented matrix =   |
| Mathematical behaviours | Marks |
| * correctly transfers coefficients of equations to augmented matrix
 |  1 |

**Question 2 (b) (3 marks)**

|  |
| --- |
| Solution |
| After and the system is reduced to:  After the system is further reduced to  |
| Mathematical behaviours | Marks |
| * correctly reduces *x*-components to 0 for rows 2 and 3 (or equivalent)
* correctly reduces *y*-component to 0 for row 3 (or equivalent)
 |  2 1 |

**Question 2 (c) (3 marks)**

|  |
| --- |
| Solution |
| From the augmented matrix form we deduce that (i) for unique solution, (ii) for no solution and (iii) for infinitely many solutionsand  |
| Mathematical behaviours | Marks |
| * correctly determines value of  for a unique solution
* correctly determines values of  and  that means there is no solution
* correctly states the values of  and  for infinitely many solutions
 |  111 |

**Question 2 (d) (3 marks)**

|  |
| --- |
| Solution |
| When and the augmented matrix becomes Then the second equation gives and first equation then leads to Hence the general solution of the equations is  |
| Mathematical behaviours | Marks |
| * determines in terms of (or vice-versa)
* determines the value of
* states the general solution in terms of a suitable parameter
 |  111 |

**Question 3 (5 marks)**

|  |
| --- |
| Solution |
| If then so that  For this to be the same as the linear function then comparison of the coefficients of and the constant requires that  and Hence  so . If  then  so  If  then  is arbitraryWe conclude that either or with any real number |
| Mathematical behaviours | Marks |
| * derives equation for the inverse
* compares coefficients to determine the equations for and
* solves for
* derives correct solution for
* dervies correct solution for
 | 11111 |

**Question 4 (4 marks)**

|  |
| --- |
| Solution |
|  Let in which case ; additionally Now  as required |
| Mathematical behaviours | Marks |
| * writes down an appropriate form for and hence
* derives an expression for
* in quotient multiplies through by the complex conjugate
* draws a valid conclusion
 | 1111 |

**Question 5 (a) (3 marks)**

|  |
| --- |
| Solution |
|  is defined if If then  if If then  if So is defined for  |
| Mathematical behaviours | Marks |
| * obtains positivity requirement for
* obtains lower and upper limits of the domain
 | 11+1 |

**Question 5 (b) (3 marks)**

|  |
| --- |
| Solution |
|  |
| Mathematical behaviours | Marks |
| * displays general shape of the graph
* indicates maximum at
* makes clear the non-differentiability at the maximum point
 | 111 |

**Question 6 (a) (2 marks)**

|  |
| --- |
| Solution |
| print grid 3d paper free2 |
| Mathematical behaviours | Marks |
| * correctly sketches the triangle on the plane with all three intercepts cutting the axes at  (-1 for one mistake)
 |  2 |

**Question 6 (b) (1 mark)**

|  |
| --- |
| Solution |
| Substituting gives: Now LHS so the given point lies on the plane |
| Mathematical behaviours | Marks |
| * correctly substitutes point into equation and confirms value
 |  1 |

**Question 6 (c) (2 marks)**

|  |
| --- |
| Solution |
| The vector **q** = 4**i****j****k** is perpendicular to *Q*. Since **v** **q**, it can be concluded that **v** is parallel to **q**As such **v** must also be perpendicular to *Q*. |
| Mathematical behaviours | Marks |
| * recognises that **v** is a scalar multiple of **w**
* concludes that **v** is parallel to **w** and so must also be perpendicular to *Q*
 |  1 1 |

**Question 6 (d) (2 marks)**

|  |
| --- |
| Solution |
| Equation for so that  |
| Mathematical behaviours | Marks |
| * writes down equation with same coefficients
* shows how to incorporate the fact that the required plane includes the given point
 |  1 1 |

**Question 6 (e) (2 marks)**

|  |
| --- |
| Solution |
| We can find **w** by forming the vector productThis vector, or any non-zero multiple of it, is the required perpendicular vector.  |
| Mathematical behaviours | Marks |
| * makes clear the need to construct a vector product
* computes the vector product correctly
 |  1 1 |

**Question 7 (6 marks)**

|  |
| --- |
|  Solution |
|  First note that   Then  for  by de Moivre’s theorem Hence the five roots are  where  Restricting the argument to the stated domain leaves  |
| Mathematical behaviours | Marks |
| * writes in a suitable polar form (1 for modulus, 1 for argument)
* uses de Moivre’s theorem appropriately
* writes down the five required roots (-1 for one mistake)
* calculates all the arguments so that they lie in the appropriate given range
 | 1+1121 |

**Question 8 (6 marks)**

|  |
| --- |
| Solution |
| The graph of is obtained by shifting the graph of 3 units to the right.The graph of is obtained by shifting the graph of 3 units down.The graph of is the same as the graph of for and then that part is reflected across the y-axis. |
| Mathematical behaviours | Marks |
| * displays the correct geometric transformations
* plots the graphs reasonably accurately
 | 1+1+11+1+1 |